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Thermodynamics of Natural Selection

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It is shown that biological-natural-selection-like behavior can occur, as a general type of time evolution, in a statistical system where detailed balance is violated owing to the presence of metastable energy states. A model of a nonequilibrium phase transition corresponding to the spontaneous origin of self-reproduction in the system is suggested. After a phase transition, the system passes from one quasistationary distribution of self-reproducing subsystems to another, with an increase in the total organization, as long as the growth of the energy flow through the system or a reduction of energy dissipation in the system is possible. The entropy production is calculated for this process in terms of "selective values" of Eigen's theory for self-organization in autocatalytic systems. Correspondence of the extremal principle of Eigen's theory with the criterion of evolution in Prigogine's thermodynamics is established.

KEY WORDS: Nonequilibrium thermodynamics; biological evolution; natural selection; self-organization; autocatalytic systems.

1. INTRODUCTION

Recently it has been realized that processes of self-organization in different nonequilibrium systems reveal common features which can be described using similar mathematical methods.[1-3] It concerns such unconnected problems as growth of instabilities in semiconductors and laser systems,[3] Benard's problem in hydrodynamics, concentration waves in chemical reactions,[1,2] etc. In all these cases a statistical many-body system is investigated under external conditions which prevent thermodynamic equilibrium. The macroscopic state of the system is considered depending on some external parameter p (or on a set of parameters) causing a deviation from thermodynamic equilibrium. For small p this deviation is also small and the state of the system does not differ qualitatively from the equilibrium state. When p reaches some critical value, a certain type of

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statistical fluctuation in the system turns from decay to amplification until a new stationary state is established. In this case the system reveals new macroscopic behavior which is characterized by a new kind of time or spatial order. As a rule, stability of the new state also has an upper limit in the parameter p above which the system turns into another stationary state, and so on. Such phenomena, which were called nonequilibrium phase transitions, in many cases show the same similarity as is characteristic for equilibrium phase transitions.

There are two reasons for this qualitative similarity on the macroscopic level. The first one is of a mathematical nature. In terms of kinetic equations describing the macroscopic behavior of a nonequilibrium system, the nonequilibrium phase transition corresponds to a bifurcation of solutions of nonlinear equations at certain values of external parameters. Universal behavior of solutions near the bifurcation point[4] is responsible for the mathematical similarity in the description of different nonequilibrium phase transitions.

The second reason stems from the extremal principle of nonequilibrium thermodynamics which is valid near equilibrium.[5,1] Any nonequilibrium system is characterized by flows of energy and matter through the system, and, therefore, by inevitable dissipation of free energy into heat. The extremal principle defines the stationary state of the system as characterized by the minimal energy dissipation at given external conditions causing energy and matter flow through the system. In a stable stationary state the excess entropy production resulting from any fluctuation is positive. Instability can occur at some values of external parameters if fluctuations reducing the entropy production can appear in the system. This gives a universal criterion of evolution for nonequilibrium systems,[2] in the spirit of Boltzmann's H theorem in equilibrium thermodynamics.

A common approach to different nonequilibrium problems and a search for general evolutionary principles of nonequilibrium thermodynamics are, to a great degree, stimulated by a desire to understand in physical terms the process of biological evolution. The latter is characterized by a long, continuous increase of the total organization of matter on the surface of the Earth. Self-reproduction, mutations, and selection, as well as an increasing consumption of free energy, are also characteristic features of this process. Though the evolutionary process is incomparably more complicated than nonequilibrium processes studied in laboratories, it may be assumed that it follows general physical law.

An approach which, to a certain extent, could be considered as a first approximation necessary for understanding the evolutionary process was developed by Eigen.[6,7] Eigen has considered a set of macromolecular information carriers which are assumed *a priori* to instruct their own syn-

thesis. Reproduction of each macromolecular species is coupled via mutations with reproduction of other species. Selection occurs in the system as a result of certain constraints imposed on the total population. A mathematical description of this process[6–9] reveals general features of biological natural selection. It also shows that time evolution of the system can be characterized by the extremal principle. The system approaches a stationary distribution of macromolecular species which corresponds to a maximal rate of their self-consistent reproduction. The stationary distribution of species represents a stable biological population. This thermodynamically nonequilibrium state of the system was called "selection equilibrium." Besides rapid evolution to the selection equilibrium in the system, there can also be a slow evolution which proceeds via rare mutations giving birth to new macromolecular species with a large reproduction rate. Such species become amplified in number until a new stationary distribution of macromolecular species is established. The new selection equilibrium is characterized by a greater rate of self-consistent reproduction than the previous one.

As was pointed out by Eigen,[6] mutations which disturb selection equilibrium correspond in Prigogine's nonequilibrium thermodynamics to fluctuations reducing the entropy production. Although this analogy is extremely important for understanding the evolutionary process, its quantitative representation has not been done yet. Moreover, analysis of Eigen's model shows that time evolution in autocatalytic systems is aimed at the increase of energy and matter consumption in the system.[2] At least early stages of this process show a tendency toward an increase of energy dissipation. It is clear, however, that if external constraints are imposed on the total energy flow through the system, then its later evolution should require a reduction of the energy dissipation in accordance with Prigogine's criterion of evolution. This is in agreement with laboratory experiments on microevolution (see, e.g., Refs. 10, and 11) as well as with traditional ideas of bioenergetics.[12] It should be pointed out, however, that there is no *a priori* quantitative correspondence between Eigen's and Prigogine's extremal principles. The minimum entropy production theorem[5] holds only close to equilibrium. In this range self-organization and evolution are excluded. Nevertheless, the possibility of developing a common thermodynamical approach to autocatalytic processes is worth exploring. These questions are discussed in the present paper in the framework of a statistical approach.

Section 2 discusses self-organization via self-reproduction in a statistical system in terms of phase trajejctories of subsystems. Amplification of a certain type of statistical fluctuations in the system is treated as attraction of phase trajectories of subsystems, owing to their

specific interaction, into a certain region of phase space. A master equation is derived for this process. Reduction of the entropy of the system is considered as a measure of the total organization.

Section 3 studies time evolution of the total organization and distribution of subsystems in the attracting region of the phase space. Eigen's selection equations are obtained from the master equation as a result of turning to an incomplete description of the system. The origin of self-reproduction is considered as a nonequilibrium phase transition which occurs in the system after the phase trajectory of a single subsystem accidentally comes into a certain region of the phase space. It is shown that the increase of the "selective value" established for autocatalytic processes by Eigen, also leads to the increase of the total organization in the phase space of the system.

Section 4 calculates the energy flow through the system which is necessary to maintain self-organization via self-reproduction. The evolution of the self-reproducing process is aimed at the increase of the total organization. Early stages of this process are shown to be characterized by the growth of the energy flow through the system. When the energy flow reaches its maximal value allowed by external constraints, the system follows Prigogine's criterion of evolution. We obtain the entropy production for the self-reproducing process in terms of Eigen's "selective values" and show that mutation, which in Eigen's theory increases selective value, corresponds in Prigogine's thermodynamics to the fluctuation which decreases the entropy production.

Section 5 is devoted to a discussion of the relation of our model to natural self-reproducing processes. The presence of metastable energy states is considered as the reason for self-organization. A certain type of statistical fluctuation can become amplified if it makes lower potential barriers for metastable energy states, resulting in a stationary self-organization via self-reproduction, which is maintained by the flow of free energy from metastable states. Metastability also breaks the detailed equilibrium in the system, which allows for the evolution from one quasistationary self-organized state to another one characterized by a greater energy flow through the system.

2. SELF-ORGANIZATION VIA SELF-REPRODUCTION IN STATISTICAL SYSTEMS

As is known. the presence of organization in a stationary state of a closed macroscopic system can be expressed in terms of the entropy of the system

$$S = -\sum_i p_i \ln p_i \tag{1}$$

Here p_i is the probability of finding the system in a microscopic state characterized by a set of parameters $\{i\}$, $\sum_i p_i = 1$. Such a microstate might for example specify the type and box location of each molecule when position-velocity space is decomposed into a fine mode of boxes. If external conditions are consistent with establishing thermodynamic equilibrium in the system, then the H theorem restricts its time evolution to the increase of the entropy with some additional constraints imposed on macroscopic parameters of the system. For an arbitrary initial state the entropy of the system approaches a maximum in a finite relaxation time. In terms of the phase space of the system, this corresponds to a random walk of the phase trajectory of the system between microstates $\{i\}$ which are allowed by the constraints on the macroscopic parameters. In the limit $t \to \infty$ the phase trajectory uniformly covers the allowed area of the phase space and passes through every microstate. As a result, the equilibrium state of the system is characterized by the same probability of all microstates, $p_i = \Omega^{-1}$, where Ω is the total number of allowed microstates, i.e., the statistical weight of the system. The maximal value of the entropy is given by $S = \ln \Omega$.

In a stationary nonequilibrium state the entropy does not take the maximal value. The presence of organization in the system means that its phase trajectory is confined in the area of the phase space ω which is less than the phase volume Ω allowed by external constraints. This situation corresponds to self-organization if it is maintained during arbitrarily long time. Otherwise we would be dealing with the approach to an equilibrium or nearly equilibrium state from a specially prepared nonequilibrium state. If probabilities of different microstates within ω are the same for all microstates, then the level of self-organization can be characterized by the value

$$I_\omega = \ln \frac{\Omega}{\omega} \tag{2}$$

which gives the decrease of the entropy in the system. The behavior of the phase trajectory described above is well known for dynamical systems where it corresponds to the presence of an attractor in the phase space of the system.[4] Of course not all attractors correspond to organized states, far from it, but we assume for the present that ω does correspond to such states in a sense to be discussed further later on.

To understand how this situation can occur in a statistical system, let us divide the system into approximately identical small subsystems and consider an ensemble of phase trajectories of subsystems. All molecules in a spatial subvolume might represent such a subsystem. In view of the fact that interacting subsystems of a larger macroscopic system cannot be

chosen absolutely identical, they are characterized by different sets of microstates. We assume that subsystems can be chosen approximately identical in the sense that for every microstate of every subsystem there are very close microstates of other subsystems, so we have no practical possibility of distinguishing them. Indeed, the total number of microstates for even a very small subsystem with a macroscopic number of degrees of freedom A is or the order of $\exp A$, which is astronomically large. The possibility of investigating phase trajectories of subsystems in a common phase space is then reasonable if large enough cells of the phase space (including macroscopic equal numbers of microstates) are considered instead of separate microstates. A coarse but typical cell specification would be the set of numbers of molecules of various types.

Now let $\{i\}$ enumerate cells of the subsystem phase space and Ω be the phase volume of a subsystem. For an ensemble of system trajectories, the probability $p_i(t)$, defined as the fraction of subsystems in the cell $\{i\}$ of the phase space, satisfies

$$\dot{p}_i(t) = \sum_{j \in \Omega} w_{ij} p_j - \sum_{j \in \Omega} w_{ij} p_i, \qquad \sum_{i \in \Omega} p_i = 1 \tag{3}$$

where w_{ij} is the probability per unit time for a subsystem to pass from the cell $\{j\}$ into the cell $\{i\}$. The first and second terms on the right-hand side of Eq. (3) correspond to phase trajectories coming into and leaving the cell $\{i\}$, respectively. A stationary homogeneous distribution of subsystems over the phase space (when all p_i are the same) is possible when microscopic reversibility is present in the system, i.e., $w_{ij} = w_{ji}$. This case usually appears when considering the equilibrium state, where the interaction between subsystems is not essential. If we consider the more general case of interacting subsystems of a large nonequilibrium system, the correlation between phase trajectories of subsystems should be taken into account. In this case the probability w_{ij} itself depends on the distribution of subsystems over the cells of the phase space, and Eq. (3) becomes nonlinear. We will assume that the behavior of every subsystem at a given moment of time depends only on the states of other subsystems at the same moment of time. This means that w_{ij} can depend on time only through variables $p_i(t)$. This dependence remains uncertain, however, until we make additional assumptions as to the form of the interaction between subsystems. The latter depends on the manner in which the system is divided into small subsystems.

In terms of phase trajectories of subsystems, the presence of self-reproduction in the system corresponds to the following dynamics. Some of the subsystems are organized, i.e., their phase trajectories are confined in the area of the phase space $\omega < \Omega$ during a certain lifetime. After the

lifetime is over, the trajectory of a subsystem leaves ω and, then, randomly walks over the phase space Ω, which corresponds to the transition of the subsystem into a disorganized state. At the same time, subsystems present in ω self-reproduce by ordering disorganized subsystems. This process results from the specific interaction which subsystems being in ω have with other subsystems. It corresponds to the attraction of phase trajectories of disorganized subsystems into ω. Note that we do not consider ω itself to be an attractor in the sense used in dynamical systems. In our model ω receives the attracting property only after at least a single phase trajectory enters into ω. We shall nevertheless continue, for simplicity, to refer to it as attracting region. We assume the statistical weight of ω to be so small in comparison with Ω that the probability of the accidental entrance of the phase trajectory of a subsystem in ω is negligible in comparison with the probability to be attracted into ω. For the same reason we will neglect the probability of any transition between cells of the attractor ω, assuming it to be very small in comparison with the probability to leave ω.

Let us now turn to the incomplete description of the system which is concerned only with the attracting region of the phase space ω. The total probability for a subsystem to be in ω is defined by

$$r(t) = \sum_{i \in \omega} p_i(t) < 1 \tag{4}$$

Then the probability to be out of ω is $1 - r(t)$. We consider any disorganized subsystem to be close to thermodynamic equilibrium, and we can certainly imagine the cells defined so that the probability of the occupation of a cell $\{j\}$ which does not belong to ω is approximately the same for all j,

$$p_j = \frac{v}{\Omega} [1 - r(t)] \tag{5}$$

where v is the number of microstates in a single cell of the phase space. Substituting Eq. (5) into Eq. (3), and taking into account all the above assumptions leads us to believe that our system will behave in a way which can be described by a master equation for cells $\{i\}$ of the attractor ω only:

$$\dot{p}_i = \frac{v}{\Omega} [1 - r(t)] \, w_i - D_i p_i \tag{6}$$

Here, $w_i = \sum_{j \in \Omega} w_{ij}$ is the probability for a phase trajectory being out of ω to be attracted into the cell $\{i\}$ of ω, $D_i = \sum_{j \in \Omega} w_{ji}$ is the probability to leave ω from the cell $\{i\} \in \omega$. The simplest model for the catalytic process

of attraction of phase trajectories into ω corresponds to the following form of w_i:

$$w_i = \sum_{j \in \Omega} w_{ij} = \frac{\Omega}{v} \sum_{k \in \omega} V_{ik} p_k \tag{7}$$

where V_{ik} is the probability per unit time for a subsystem in the cell $\{k\} \in \omega$ to attract a phase trajectory of disorganized subsystems into the cell $\{i\} \in \omega$. After substitution of Eq. (7) into Eq. (6) we get

$$p_i = \sum_{k \in \omega} L_{ik} p_k \tag{8}$$

where

$$L_{ik} = V_{ik}[1 - r(t)] - D_i \delta_{ik} \tag{9}$$

The quantities D_i give inverse lifetimes of phase trajectories within ω. Note that since the cells of the phase space still contain macroscopic numbers of microstates, the behavior of subsystems within the attractor, to some degree, remains uncertain.

The value $r(t)$ gives the average fraction of organized subsystems. It can be used, therefore, as a measure of self-organization in the system, as well as the quantity I_ω. The latter quantity, contrary to $r(t)$, is a parameter of the model, since we assume *a priori* the existence of the attracting region in the phase space of the system. The presence of such a region in the phase space can be considered a necessary condition for the possibility of self-organization via self-reproduction, while $r(t)$ represents the realized level of self-organization.

We would like to emphasize that our statistical model is a direct realization of Prigogine's idea of "order through fluctuations." Indeed, amplification of a certain type of statistical fluctuations in the system can be considered in terms of the attraction of phase trajectories of subsystems, due to their specific interaction, into a certain region of the phase space. Thus the attractor in the above sense is always present in nonequilibrium systems which possess a nonequilibrium phase transition.

3. SELECTION IN THE PHASE SPACE

We are interested in the total organization $r(t)$ and in the distribution of subsystems over the cells of the attractor. This distribution is expressed in terms of conditional probabilities

$$x_i(t) = \frac{p_i(t)}{r(t)} \tag{10}$$

Substituting $p_i = rx_i$ into Eq. (8) we obtain

$$\dot{x}_i + x_i \frac{\dot{r}}{r} = \sum_k L_{ik} x_k \tag{11}$$

According to Eq. (4), the new variables x_i satisfy the condition

$$\sum_i x_i(t) = 1 \tag{12}$$

It gives the following equation for $r(t)$:

$$\frac{\dot{r}}{r} = \sum_{i,k} L_{ik} x_k \tag{13}$$

Substituting Eq. (13) into Eq. (11), we have

$$\dot{x}_i = \sum_k L_{ik} x_k - x_i \sum_{j,k} L_{jk} x_k \tag{14}$$

The second term in Eq. (14) has appeared owing to our turning to the incomplete description of the system in terms of conditional probabilities x_i. Equation (14) is the same equation that appears in Eigen's theory for the selection process in a collection of self-reproducing macromolecular information carriers.[7] In this theory the second nonlinear term in Eq. (14) has been obtained under a certain environmental constraint imposed on the total population. In particular, it can be obtained when the dilution flow for a macromolecular species is taken to be proportional to its concentration and the proportionality is assumed to be the same for all species. Our consideration shows that the selection equation (14) has a much greater generality. In statistical systems it describes the process of self-reproduction and self-organization irrespective of environmental constraints. It does not even require the system to consist of a finite number of subsystems. For an infinite system the factor $(1-r)$ in the first term of Eq. (9) must be replaced by 1. The selection equation (14) for an infinite system, then, holds even if the second term in Eq. (9) corresponding to the destruction of organized states is omitted, though in this case the system possesses unlimited growth of the organization. Thus the only assumption which leads to Eq. (14) is the assumption that a certain type of statistical fluctuation can become amplified in the system.

In fact we deal with a system of nonlinear Eqs. (12)–(14), since the functions L_{ik} depend on $r(t)$ according to Eq. (9). We will give a solution of these equations for the case when the lifetime for a subsystem in organized

state is the same for all cells of the attractor, $D_i = D$. In this case, Eqs. (13), (14) with account of Eq. (10) take the form

$$\dot{r} = r(1-r)\sum_{i,k} V_{ik} x_k - Dr \tag{15}$$

$$\dot{x}_i = (1-r)\left(\sum_k V_{ik} x_k - x_i \sum_{j,k} V_{jk} x_k\right) \tag{16}$$

It is convenient to use the new time variable

$$\tau(t) = \int_0^t [1 - r(t')]\, dt' \tag{17}$$

instead of t. In terms of τ Eqs. (15), (16) can be written as

$$\frac{dr}{d\tau} = r\sum_{i,k} V_{ik} x_k - \frac{Dr}{1-r} \tag{18}$$

$$\frac{dx_i}{d\tau} = \sum_k V_{ik} x_k - x_i \sum_{j,k} V_{jk} x_k \tag{19}$$

To find solutions of these equations let us introduce new probability variables[8,9]

$$x_i = \sum_k C_{ik}\, y_k, \qquad \sum_i y_i = 1 \tag{20}$$

This implies

$$\sum_i C_{ik} = 1 \tag{21}$$

Coefficients C_{ik} are defined as the matrix which reduces the matrix V_{ik} to the diagonal form

$$(\hat{C}^{-1}\hat{V}\hat{C})_{ij} = \lambda_i \delta_{ij} \tag{22}$$

Here λ_i are the eigenvalues of V_{ik}. It is easy to see from Eqs. (21), (22) that C_{ik} is the ith component of the eigenvector belonging to the eigenvalue λ_k, and also that

$$\lambda_k = \sum_{j,i} V_{ji} C_{ik} \tag{23}$$

For the matrix of rank n, Eq. (21) gives n conditions on C_{ik} and Eq. (22) gives $n^2 - n$ conditions. Both Eqs. (21), (22) give n^2 conditions and hence

completely define the coefficients C_{ik}. In terms of the new variables and with the help of Eqs. (20)–(23) we can rewrite Eqs. (18), (19) in the form

$$\frac{dr}{d\tau} = r \sum_i \lambda_i y_i - \frac{Dr}{1-r} \tag{24}$$

$$\frac{dy_i}{d\tau} = \left(\lambda_i - \sum_k \lambda_k y_k \right) y_i \tag{25}$$

We will follow Eigen's fundamental work[7] in analysis of the selection process. The sum $\sum_k \lambda_k y_k$ corresponds to averaging the eigenvalues over y_k. Though λ_k and y_k may be complex, $\sum_k \lambda_k y_k$ is real because there are conjugates of every term in the sum. The latter follows from the reality of the matrix V_{ik} which defines the transition probabilities.

According to Eq. (25) the variable y_i increases in magnitude if $\mathrm{Re}\,\lambda_i > \sum_k \lambda_k y_k$ and decreases if $\mathrm{Re}\,\lambda_i < \sum_k \lambda_k y_k$. This selection process, in its turn, causes the increase of the mean eigenvalue $\sum_k \lambda_k y_k$. Thus y_i corresponding to greater and greater $\mathrm{Re}\,\lambda_i$ are selected in the process. At the same time, the total organization defined by the variable $r < 1$ increases until the second term in Eq. (24) becomes large. The probability distribution x_i can be obtained after substitution of selected variables y_i into Eq. (20). According to Eigen[7] this distribution represents a natural biological population of coupled self-reproducing information carriers.

Solutions of Eqs. (24), (25) can be written

$$r = r(0) \sum_i y_i(0) \exp(\lambda_i \tau - Dt) \tag{26}$$

$$y_i = \frac{y_i(0) \exp \lambda_i \tau}{\sum_k y_k(0) \exp \lambda_k \tau} \tag{27}$$

With the help of Eqs. (17) and (20), we then finally obtain

$$r(t) = r(0) \sum_i y_i(0) \exp \int_0^t \{\lambda_i [1 - r(t')] - D\}\, dt' \tag{28}$$

$$x_i(t) = \frac{\sum_k C_{ik}\, y_k(0) \exp \lambda_k \int_0^t [1 - r(t')]\, dt'}{\sum_k y_k(0) \exp \lambda_k \int_0^t [1 - r(t')]\, dt'} \tag{29}$$

Parameters $y_i(0)$ are defined as

$$y_i(0) = (\hat{C}^{-1})_{ik}\, x_k(0) \tag{30}$$

where $x_k(0)$ gives the initial probability distribution within the attracting

region of the phase space. Since the selecting process begins with a single organized subsystem, the initial probability distribution can be chosen as

$$[x_i(0)] = (1, 0, 0, \ldots, 0) \tag{31}$$

The parameter $r(0)$ represents the probability of starting the evolution, i.e., the trajectory of a subsystem within the attracting region of the phase space. We consider this probability to be extremely small, so $r(0)$ is a very small number.

The matrix V_{ik} is nonnegative by definition. Further analysis of the evolutionary process is based on the remarkable Perron–Frobenius theorem for nonnegative matrices.[13] A nonnegative irreducible matrix V_{ik} has a real, positive, and nondegenerate eigenvalue λ_* (Perron number) which satisfies the condition

$$\lambda_* > |\lambda_l| \tag{32}$$

where λ_l are all other eigenvalues of V_{ik}. The eigenvector C_{i*} belonging to λ_* is the only positive eigenvector of the matrix V_{ik}. The lower and the upper bounds on λ_* are defined by the inequalities

$$\min_k \sum_i V_{ik} < \lambda_* < \max_k \sum_i V_{ik} \tag{33}$$

Note that $\sum_i V_{ik}$ gives the total probability per unit time for the subsystem in the cell $\{k\} \in \omega$ to attract into ω the phase trajectory of a disorganized subsystem. Hence, the lower and the upper bounds on λ_* are defined by the minimal and the maximal catalytic activity of cells of the attractor.

Using the Perron–Frobenius theorem, we obtain from Eqs. (28) and (29) the following solution for the stationary state of the system:

$$r = 1 - \frac{D}{\lambda_*} \tag{34}$$

$$x_i = C_{i*} \tag{35}$$

It shows that the stationary autocatalytic process can be established in the system due to the inequality $\lambda_* > D$. After this condition is satisfied, the disorganized state of the system becomes unstable against spontaneous origin of self-reproduction. The probability of the fluctuation leading to this process is given by $r(0)$. The transition of the system from the nearly equilibrium state to the stationary process of self-reproduction can be considered a nonequilibrium phase transition. The order parameter for this transition is given by Eq. (34).

If the nonnegative matrix V_{ik} is reducible, i.e., of the form

$$V_{ik} = \left(\begin{array}{c|c} V_{ik}^{(1)} & 0 \\ \hline 0 & V_{ik}^{(2)} \end{array}\right) \tag{36}$$

then the evolution of the system depends on the initial condition (31). It is defined by the matrix $V_{ik}^{(1)}$ or $V_{ik}^{(2)}$ depending on which cell of the attractor was initially occupied. Correspondingly, Perron number $\lambda_*^{(1)}$ or $\lambda_*^{(2)}$ defines the stationary state of the system. The matrix of the form (36) corresponds to the splitting of the attracting region of the phase space ω into two independent attractors ω_1 and ω_2. Let us now assume that the evolutionary process begins with ω_1 and that there is very small but nonzero probability of the catalysis $\omega_1 \to \omega_2$, where $\lambda_*^{(1)} < \lambda_*^{(2)}$. This means that some subsystems, whose phase trajectories are confined within ω_1, have now nonzero probability to attract phase trajectories of disorganized subsystems into ω_2. It is not difficult to obtain from Eqs. (28) and (29) what will be the evolution of the system in this case. A fluctuation corresponding to a single event of the catalysis $\omega_1 \to \omega_2$, breaks the stability of the first stationary state characterized by the selection equilibrium $x_i^{(1)} = C_{i*}^{(1)}$ and by the order parameter $r_1 = 1 - (D/\lambda_*^{(1)})$. This fluctuation becomes amplified until the new selection equilibrium $x_i^{(2)} = C_{i*}^{(2)}$ and the new order parameter $r_2 = 1 - (D/\lambda_*^{(2)}) > r_1$ are established in the system. The characteristic time of this process is $(\lambda_*^{(2)} - \lambda_*^{(1)})^{-1}$, while the time of waiting for the fluctuation is defined by its probability and may be much greater.

Generally, the attracting region of the phase space can be split into a large number of weakly connected blocks $\omega_1, \omega_2, \ldots, \omega_m$. In this case the evolution of the system consists of alternating rapid and slow stages. A rapid stage corresponds to establishing the selection equilibrium in a certain block, while the slow evolution proceeds via rare transitions from one block to another with increasing Perron number,

$$\lambda_*^{(1)} \to \lambda_*^{(2)} \to \cdots \to \lambda_*^{(\max)} \tag{37}$$

Distribution functions $x_i^{(1)} = C_{i*}^{(1)}$, $x_i^{(2)} = C_{i*}^{(2)}, \ldots, x_i^{(m)} = C_{i*}^{(m)}$ are consecutively realized in this process.

In Eigen's theory these distribution functions represent different biological populations which originate from each other due to mutations. The value $\lambda_*^{(n)} - D$ corresponds in Eigen's theory to the "selective value" of the population $x_i^{(n)}$. Our consideration shows that the increase of the "selec-

tive value" established for autocatalytic processes by Eigen, also leads to the increase of the total organization in the phase space of the system,

$$r_1 \to r_2 \to \cdots \to r_{\max} \tag{38}$$

where $r_n = 1 - (D/\lambda_*^{(n)})$.

4. THERMODYNAMICS OF THE EVOLUTIONARY PROCESS

Let N be the total number of subsystems in the system. Then, the self-organized system can be considered as consisting of two large parts. One of them includes rN organized subsystems, while another part includes $(1-r)N$ disorganized subsystems. It should be noted, however, that since we use the incomplete description of the system, the division of its disorganized part into small subsystems is conventional. Indeed, studying the cyclic proces of reproduction and destruction of organized states, we should not consider phase trajectories of subsystems coming into ω as belonging to the same subsystems which formerly left ω. In our model the disorganized part of the system is a nearly equilibrium environment for the organized part of the system. A certain subsystem is singled out from this environment just when its phase trajectory occurs in the attracting region of the phase space. It becomes possible owing to a specific interaction between organized subsystems and the disorganized environment. In this sense the autocatalytic process is similar to the process of measurement in quantum mechanics. As will become clear from the analysis of the informational aspect of the problem, this analogy proves to be rather deep.

The rate of the autocatalytic process is defined by the interaction between organized subsystems and the disorganized environment. We assume that the process is slow enough to consider the interaction as very weak. In this case the total entropy of the system can be defined as

$$\mathrm{S_t} = (1-r)\,NS_\Omega + rNS_\omega \tag{39}$$

where S_Ω is the entropy of a disorganized subsystem, S_ω is the entropy of a subsystem whose phase trajectory is confined with ω. The total entropy can be represented also as

$$S_t = S_e - I \tag{40}$$

where $S_e = NS_\Omega$ is the total equilibrium entropy of the system, I is the decrease in the total entropy resulting from self-organization,

$$I = rN(S_\Omega - S_\omega) \tag{41}$$

Note that in the theory of information, I is the quantity of the information which the observer receives about the system due to its self-organization. Correspondingly, $S_\Omega - S_\omega$ is the quantity of the information created in a single act of self-reproduction, i.e., when the phase trajectory of some subsystem occurs within ω. We can express these quantities in terms of variables which define the autocatalytic process. Let M and m be the total numbers of cells in Ω and ω, respectively, and v be the number of microstates in every cell of the phase space. Then S_Ω is defined as

$$S_\Omega = \ln vM = \ln \Omega \tag{42}$$

The probability for the organized subsystem to be in a microstate belonging to the cell $\{i\} \in \omega$ is equal to x_i/v. Thus the entropy of the organized subsystem is given by

$$S_\omega = -\sum_{i=1}^{m} x_i \ln \frac{x_i}{v} \tag{43}$$

If the probability x_i were the same for all cells of ω, then Eq. (43) would give

$$S_\omega = \ln vm = \ln \omega \tag{44}$$

In our model the stationary distribution of probabilities within ω is given by Eq. (35) and S_ω takes the form

$$S_\omega = \ln v - \sum_{i=1}^{m} C_{i^*} \ln C_{i^*} \tag{45}$$

For the stationary state of the system, with the help of Eqs. (34), (42), and (45), we obtain

$$I = \left(1 - \frac{D}{\lambda_*}\right) N \left(\ln M + \sum_{i=1}^{m} C_{i^*} \ln C_{i^*}\right) \tag{46}$$

In the stationary state I remains constant up to fluctuations of the order parameter r and the probability distribution x_i. In this case the decrease of the entropy in the reproduction processes is compensated by the same increase of the entropy resulting from the destruction of information carriers. In the course of a single act of reproduction, a small subsystem of nearly equilibrium disorganized environment becomes ordered, so its entropy decreases by the value $S_\Omega - S_\omega$. According to the first law of thermodynamics,[14] the minimal work

$$W_{\min} = T(S_\Omega - S_\omega) \tag{47}$$

must be done in this process, where T is the constant temperature of the system. This work is qual to the increase of the free energy of a subsystem in the organized state. Equation (47) also gives the maximal work which can be done by the organized subsystem during its transition into a disorganized state. We will first assume that the free energy realized from the destruction of the information carrier cannot be used for the process of self-reproduction, though there is another possibility which will be discussed below. Then the stationary process of reproduction and destruction of information carriers needs the source of free energy. It is easy to see that its minimal power is given by

$$\left(\frac{dF}{dt}\right)_{\min} = DTI \tag{48}$$

where D is the inverse lifetime of the information carrier. It is also clear that the process is possible if the system is able to use the free energy for doing the work to order its own subsystems. In our consideration this condition coincides with the ability of subsystems, being in a certain region of the phase space, to attract into this region the phase trajectories of other subsystems. Here we do not dwell on the nature of the free energy source. The correspondence of our model to natural self-reproducing processes will be discussed in the next section.

Equation (47) gives the minimal work which must be done in the ideal isothermal process of ordering an equilibrium subsystem. In the real process of reproduction only a part of the free energy coming to the system is accumulated by organized subsystems, while another part is dissipated into heat. Thus work greater than $W_{\min}$ must be done in the real process of reproduction. According to our assumption, the free energy (47) accumulated by the organized subsystem is also dissipated into heat after the destruction of the organized state. Hence, stationary self-organization via self-reproduction can be maintained only by the constant flow of energy through the system. We will assume that the total thermal energy accumulated in this process is radiated into the space surrounding the system, so the temperature of the system remains constant. For total consumption of free energy by the system with account of dissipation we obtain

$$\frac{dF}{dt} = DTI + T\left(\frac{\partial S}{\partial t}\right)_{\text{dis}} \tag{49}$$

where $(\partial S/\partial t)_{\text{dis}}$ is the entropy production in the system due to dissipative processes which accompany self-reproduction.

To draw a conclusion as to the evolution of the energy flow through the system, let us assume that $S_\omega \ll S_\Omega$. It is valid when the attracting

region of the phase space ω is exponentially small in comparison with Ω, which is in accordance with our assumption that the probability of the accidental entrance of the phase trajectory of a subsystem in ω is extremely small. According to Eqs. (40) and (41), this gives

$$I = rS_e \tag{50}$$

As it was shown in the previous section, the evolution starts when the phase trajectory of a single subsystem accidentally enters the attracting region of the phase space with Perron number λ_* exceeding the destruction rate D. It corresponds to the nonequilibrium phase transition into the organized state with the order parameter $r = 1 - (D/\lambda_*)$. As a result, information given by Eq. (50) originates in the system. This state is maintained against the loss of information due to self-reproduction which requires the minimal consumption of free energy given by Eq. (48). The further evolution proceeds via mutations. Successful mutation corresponds to the accidental transition of autocatalytic processes to the attracting block of the phase space with greater λ_*. Such mutations lead to the irreversible increase of the order parameter and total information. According to Eq. (48) this process is also characterized by the increase of the minimal flow of energy through the system. Therefore selection of successful mutation corresponds to the origin of a new species which is able to increase the rate of the use of free energy for its own reproduction.

The final result of the evolutionary process is defined by the structure of the phase space of the system (i.e., by allowed values of λ_*) and also by external limitation on (dF/dt). This limitation is irrelevant, however, for the early evolution, when the energy flow through the system is small compared to the maximal allowed flow P_0. The early evolution is, therefore, characterized by the growth of the total organization irrespective of the entropy production $(\partial S/\partial t)_{\text{dis}}$ in the system. It is not concerned about the economic use of free energy.

Another situation occurs when the energy flow through the system reaches its maximal allowed value P_0. In this case the only possibility for further evolution in the system results from mutations which decrease the entropy production and use the realized free energy for acceleration of self-reproduction. According to Eqs. (49) and (50), the entropy production for the later stages of the evolution is given by

$$\left(\frac{\partial S}{\partial t}\right)_{\text{dis}} = \frac{P_0}{T} - \left(1 - \frac{D}{\lambda_*}\right) DS_e \tag{51}$$

Equation (51) shows the direct correspondence between Eigen's theory for self-reproduction of macromolecular information carriers and Prigogine's

principle of minimal entropy production in nonequilibrium systems. Mutation which in Eigen's theory increases "selective value," i.e., mutation which increases the Perron number, corresponds in Prigogine's thermodynamics to the fluctuation which decreases the entropy production.

5. DISCUSSION AND CONCLUSION

Let us now discuss the relation of our model to natural self-reproducing processes and to processes studied by nonequilibrium thermodynamics. As is known,[1,2] the latter usually proceeds from the assumption of local thermodynamic equilibrium. This assumption, in its turn, is based on the assumption that the relaxation time for a subsystem decreases with the division of the system into smaller and smaller subsystems. As a result, small enough but still macroscopic subsystems can be considered as being in thermodynamic equilibrium. In this case the system can be characterized by the same macroscopic parameters as for equilibrium state (such as concentrations and flows of different substances, temperature, pressure, etc.) but depending on coordinates and time. It can be shown (see, e.g., Ref. 3) that the interaction between subsystems is neglected in such an approach. Local nonequilibrium thermodynamics takes into account correlations leading to the establishment of the stationary state only on a macroscopic level, i.e., in a mean-field manner. Our model of autocatalytic processes exceds the limits of this approach since we consider specific albeit weak interaction [see discussion preceding Eq. (39)] between subsystems.

The absence of the local thermodynamic equilibrium in natural processes of self-reproduction becomes evident when we consider the manner in which living systems use the free energy. The latter is always present in natural self-organized systems in the form of longliving metastable states. For our consideration the most important fact is that in a living system the rate of the energy flow from metastable states is defined not by their own relaxation times but by the processes which take place in the system. In the living cell the free energy carriers are ATP molecules. In green plants they are produced in the process of photosynthesis which is similar to the chain of electron transitions in a laser.[15] The electron is activated by sunlight from a low-energy level E_1 to a high energy level E_3, with posterior transition, in the course of chemical reactions, to an intermediate long-living level E_2 of ATP molecule. Then, in the course of chemical reactions corresponding to the synthesis of biological macromolecules, the electron goes down to the lowest level E_1 where it can be again activated by sunlight to the high-energy level E_3. For a given flow of sunlight, the rate of the free energy flow through the system is, then, defined by the rate of

the biosynthesis. In this process, the relaxation time for activated electron states depends on the functioning of the whole living cell. It does not become shorter when we turn to the consideration of a small part of the system. That is why the assumption of local equilibrium is unacceptable when studying bioenergetical processes.

Another important fact is that the detailed equilibrium is also absent in living systems, as it is absent in laser systems. Indeed, detailed equilibrium means that if the transition from one microstate to another is possible in the system, then the opposite transition is also possible. On the contrary, for the process of biosynthesis there are no paths in the phase space which would correspond to jumps of the electron over energy levels in the direction opposite to the chain of transitions described above. In particular, no one thermal transition $E_2 \to E_3$ can occur in the living cell for a period of the cyclic proces $E_1 \to E_3 \to E_2 \to E_1$ because the temperature of the system is small in comparison with the energy gap between E_2 and E_3. Absence of detailed equilibrium, together with certain requirements to the structure of the phase space, allows the existence of a number of stationary states in the system.[2,3]

Now we can specify in what systems the evolutionary process described in previous sections can become possible. Firstly, it must be the system which can be characterized by a number of stationary (or quasistationary) states. The system can pass from one stationary state to another owing to rare fluctuations. Secondly, there must be a source of the free energy which can be used for the maintaining of the stationary state. We will point out two kinds of statistical systems which can satisfy these requirements. The system of the first kind borders on the unlimited reservoir of metastable molecular (or nuclear) states. We assume that processes are possible in the system which make lower potential barriers for metastable states, so the system itself may be considered as a barrier for the flow of free energy through it. The system of the second kind is like laser systems. It has long-living metastable energy states of molecules which can be activated, in a closed chain of transitions, by the external source of radiation. The minimal chain consists of transitions between three levels $E_1 < E_2 < E_3$. The transition $E_1 \to E_3$ (as well as the opposite transition) is induced by the external source, while the transition $E_3 \to E_2$ to long-living energy level E_2 is spontaneous. The flow of free energy through the level E_2 depends on the rate of relaxation $E_2 \to E_1$. We again assume that processes are possible in the system (such as chemical reactions, flows of different substances, etc.) which can make lower potential barriers for long-living energy states or accelerate their relaxation through a certain chain of intermediate electron transitions. Thus the system of the second kind can also be considered as a barrier for the energy flow from E_2 to E_1. This flow,

however, is limited by the power of the external source of radiation, which distinguishes the system of the second kind from the system of the first kind. For systems of both kinds the detailed equilibrium is broken owing to the presence of metastable states. This makes possible their evolution from one quasistationary state to another one differing in the energy flow through the system.

A certain type of statistical fluctuation can become amplified in the above systems if it makes lower potential barriers for metastable energy states. In our model the fluctuation which breaks stability of the system and becomes amplified, is considered in terms of self-reproducing subsystems, i.e., in terms of subsystems which attract phase trajectories of other subsystems into a certain region of the phase space. Thus systems of both kinds described above can be unstable against spontaneous origin of self-organization via self-reproduction. In this case self-organization is maintained in the system owing to continuous flow of free energy from metastable energy states. In our terms nonequilibrium phase transition of that type can occur in the system if there is a region in the phase space of subsystems with Perron number exceeding the rate of destruction of the organized state. We have shown that the evolution of the system which has a number of stationary states is directed toward the increase of the total organization and energy flow through the system. For a system of the first kind the evolution to a greater energy flow is restricted only by the structure of the phase space of the system. Contrary to this case, the evolution of a system of the second kind is restricted also by the power of the external source of radiation. In accordance with results of the previous section, one can expect that the later evolution of the system of the second kind follows Prigogine's law of minimal entropy production.

It is evident that the larger and more complicated is the system, the longer the evolution it can have in presence of metastable energy states. A laser system gives an example of a system of the second kind. Long evolution from one stationary state to another is evidently impossible in this system owing to its very simple structure. Nevertheless, as it was pointed out by Haken[3] the short process of selection of electromagnetic modes in a laser follows the same equations which appear in Eigen's theory for self-reproduction of macromolecular information carriers. Our consideration shows why Eigen's equations have such universality for the description of self-organization in nonequilibrium systems. The only assumption that leads to Eigen's equations is the assumption that a certain type of statistical fluctuations can become amplified in the system. Self-organization of electromagnetic modes in a laser proves to be the simplest example of a short nonchemical evolutionary process.

We believe that our results can be useful for experiments on spon-

taneous origin of chemical evolution. What could be their generalization necessary to understand the energetics of biological evolution? Our model applies to the description of the process of self-reproduction of one population, or of a set of uncoupled populations. After the stationary process of reproduction and destruction is established, a certain part of the free energy which flows through the system is accumulated in organized subsystems. It allows us to consider them as metastable sources of free energy. Their lifetime D^{-1} was the parameter of our model since we assumed that the free energy accumulated in organized subsystems cannot be used for reproduction and dissipates after the destruction of organized states. The above treatment, however, allows us to consider the accumulation of free energy in metastable information carriers as a possibility for a new instability in the system. As a result of this instability, a new population of information carriers can originate which uses for self-reproduction the free energy accumulated in the first population. It gives way to an hierarchy of self-reproducing processes which supply each other with the free energy. We consider the origin of such a hierarchy a characteristic feature of the biological evolution and believe that also in this case the evolution is directed toward the increase of the total flow of energy through the system. Research on generalization of our model to this case is in progress.

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